Electromagnetic Backscattering Measurements by a Time-Separation Method*

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Summary—**The object of this research is to investigate the feasibility of adapting the conventional pulsed radar technique for close-range back-scattering measurements for obstacles of arbitrary shape and small scattering cross sections. The time-separation or microwave-pulse method described in this paper differs essentially from all previously used laboratory methods in that the scattered field does not mix with the incident field at the detector and is separated from it in time. The essential experimental arrangement of this method is similar to that of the CW magic-T method except that a source generating very short pulses is used instead of CW. Preliminary experimental data for thin circular metallic disks show that the pulse method is a feasible one, since the measured results are in close agreement with the theoretical values. Accurate back-scattering measurements for obstacles of arbitrary shape and small scattering cross sections should be obtainable by this method provided a short microwave pulse of high power level is available.**

Introduction

In a recent report King and Wu[1] made a summary of the available methods in the measurement of back-scattering cross sections of obstacles of various configurations. The frequency-separation or doppler-shift method[2,3] yields accurate measurements for obstacles with small scattering cross sections, but its usefulness is limited only to obstacles with rotational symmetry. The space-separation method used by Schmitt[4] may prove to be a promising method of precision when necessary refinements are made. The free-space time-separation method to be described in this paper is suitable for accurate back-scattering measurements for three-dimensional obstacles of arbitrary shape and small scattering cross sections. The essential experimental arrangement of the time-separation method is similar to that of the CW magic-T method[3] except that in the present experiment a source generating very short pulses is used instead of continuous waves. When a very short microwave pulse (of the order of forty mμsec) is used as a source, the scattered field from the obstacle in question is separated from the incident field in time and accurate measurements can be made for arbitrarily shaped obstacles of relatively small scattering cross sections. It becomes evident that in the CW magic-T arrangement it is desirable to use a power level that is sufficient to lift the level of the back-scattered signal above both the noise level of the receiving system and the level of the leakage signal due to imperfect decoupling between the E-arm and H-arm of the magic-T or hybrid junction. However, if the scattered field is mixed with the leakage field due to imperfect decoupling and the reflection field due to discontinuities of the horn in any given proportion, this proportion can not be altered by simply increasing the source power level. Under such circumstances an increase in input power level does not extend the range of accurate measurements. The microwave-pulse method does not have this limitation and the range of the lower limit of accurate measurement may be extended linearly by raising the power level of the source, as the scattered field in this case is separated from the leakage field and reflection field in time.

The series of obstacles used in testing the feasibility of the time-separation method are thin circular metallic disks. These disks have the largest back-scattering cross sections among all obstacles which have geometrical cross-sectional areas equal to those of the corresponding disks. It was necessary to use these disks as test samples because a short microwave-pulse of high-power level was not available in addition to the fact that exact theoretical results could be calculated[5] for comparison. In some respects spheres would have been more convenient as test samples because of their complete symmetry property if a source of high-power level had been available.

Description of the Apparatus

Fig. 1 shows a schematic diagram of the microwave pulse system. Measurements were made at 3675 mc because the waveguide equipment for this frequency happened to be immediately available. However, it is generally preferable to use *S*-band waveguide since a source of high-power pulses of about 40-mμsec width in this band can be obtained more easily. The whole equipment was installed on the roof of the Laboratory so that the obstacles could be placed in surroundings practically free from electromagnetic reflections.

* Manuscript received by the PGMTT, June 25, 1958; revised manuscript received, September 8, 1958. This research was performed under Air Force Contract No. AF (604) 786 between the AF Cambridge Res. Cntr. and the President and Fellows of Harvard College.

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[1] R. King and T. T. Wu, "The Reflection of Electromagnetic Waves from Surfaces of Complex Shape," Cruft Lab. Sci. Rep. No. 12, Harvard University; 1957.

[2] H. Scharfman and D. D. King, "Antenna scattering measurements by modulation of the scatterer," Proc. IRE, vol. 42, pp. 854–860; 1954.

[3] C. C. H. Tang, "Back-scattering from dielectric-coated infinite cylindrical obstacles," *J. Appl. Phys.*, vol. 28, pp. 628–633; 1957.

[4] H. J. Schmitt, "Back-Scattering Measurements with a Space-Separation Method," Cruft Lab. Sci. Rep. No. 14, Harvard Univ.; 1957.

[5] W. Andrejewski, "Die Beugung electromagnetisher Wellen an der leitenden Kreisscheibe und an der kreisförmigen Öffnung in leitenden ebenen Schrim," Ph.D. Dissertation, Westfalische Technische Hochschule, Aachen, Germany; 1952.

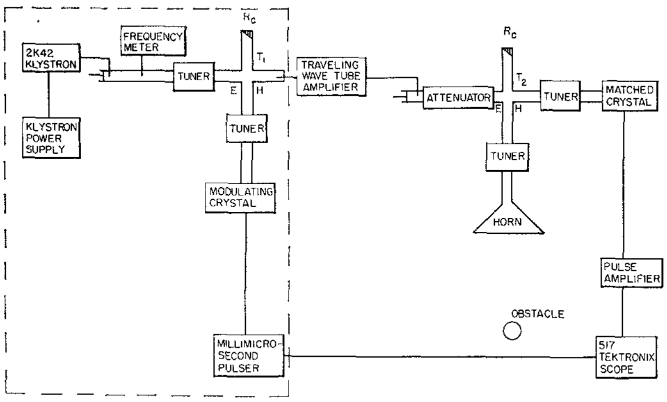

Fig. 1—Block diagram of microwave pulse system for back-scattering measurements.

The equipment in the dotted line block of Fig. 1 is used to generate a microwave-pulse with a width of about 40 mμsec. The CW magic-T method requires an extremely high decoupling between the E-arm and H-arm in order to yield reliable results. The CW magic-T method suffers the limitation that it can be used only when the magnitude of the back-scattered signal in question is considerably larger than that of the leakage signal or horn reflection signal. Since it is difficult to eliminate the small leakage, this condition imposes a lower limit on the magnitude of the back-scattered signal. The time-separation method circumvents the difficulty due to imperfect decoupling. This is accomplished by using a very short pulse so that the leakage pulse will be separated from the back-scattered pulse in time scale as viewed on a high-speed scope (such as Tektronix 517 which has a rise time of 7 mμsec).

Although it is not necessary to eliminate the leakage signal in this system, it is still highly desirable to make it as small as possible. Accordingly, the hybrid junction T_2, specially machined to a high degree of symmetry, is used instead of a magic-T, which is very difficult to make with a corresponding degree of symmetry. The symmetry in the hybrid T is obtained at the expense of narrower bandwidth.

Since the distance between the source point and the obstacle is limited by the available space in the laboratory, the width of the generated pulse must be restricted accordingly in order to prevent the overlapping of the leakage pulse and reflection pulse with the back-scattered pulse on the viewing scope. With an available separation of from 10 to 20 meters between the source and the obstacle, the maximum pulse width is in the range of 50 to 100 mμsec. Although it is desirable to use as short a pulse as possible from the standpoint of reducing the necessary distance between the source and the obstacle, it is undesirable from the point of view of frequency spread. A very broad frequency spread not only gives inaccurate scattering measurements but also difficulties of broad-band matching.

The method finally used in producing the short microwave-pulse is shown in the arrangement in the dotted block of the schematic diagram of Fig. 1. A millimicrosecond dc pulser is used to modulate the crystal in one of the side arms of the hybrid junction T_1 and a microwave pulse is generated in the fourth arm of the hybrid junction. Before the dc pulse is applied to the crystal, the tuner in front of the crystal must be so adjusted that it appears as a matched load and there is no output from the fourth arm of the junction T_1. When the dc pulse is applied to the crystal, the condition of matching is destroyed and a short microwave-pulse is produced in the fourth arm of the junction T_1. This is amplified by a traveling-wave tube and then used as the pulse source for back-scattering measurements. The pulsed output power from the horn with the present arrangement is of the order of 0.1 watt.

The rectangular dc modulating pulse used has a width of 20 mμsec. Owing to the finite response and decay times of the crystal (each of the order of a few millimicroseconds) and the rise time of 7 mμsec of the scope, the microwave pulse as observed at the input to the horn has a base width of about 40 mμsec and a top width (10 per cent down from the top) of 20 mμsec. A pulse of 40-mμsec width has a frequency spread of about 50 mc. Although the conventional slide-screw tuners are frequency-sensitive, it is found that these tuners are still fairly effective in matching such pulses. In order to preserve the original pulse shape it is necessary to avoid the use of long waveguide sections in order to prevent excessive dispersion.

Fig. 2 shows the polyfoam obstacle-stand with a disk as obstacle in place. The reflection from the polyfoam is negligible at this frequency. The turntable at the bottom can be rotated and tilted for proper orientation of the obstacle. Eleven disks were made of 3-mils copper sheet and each was glued to a polyfoam block for flatness and ease in handling. These blocks can be slid into the slot at the top of the polyfoam stand shown in Fig. 2.

Measurement and Comparison

Initial measurements were made at 3675 mc with a pulsed peak output of about 0.1 watt. The total power received by the horn from the back-scattered field is expressed by the equation[6]

$$P_r = \frac{P_i \sigma \lambda^2 G^2}{(4\pi)^3 R^4} \quad \text{or} \quad \sigma = \left(\frac{P_r}{P_i}\right) \frac{(4\pi)^3 R^4}{\lambda^2 G^2}$$

where

P_i is the total power radiated from the horn,
σ is the back-scattering cross section of the obstacle,
λ is the wavelength,
G is the gain of the horn,
R is the distance between the horn and the obstacle.

[6] S. Silver, "Microwave Antenna Theory and Design," M.I.T. Rad. Lab. Ser., vol. 12, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 12, p. 5; 1949.

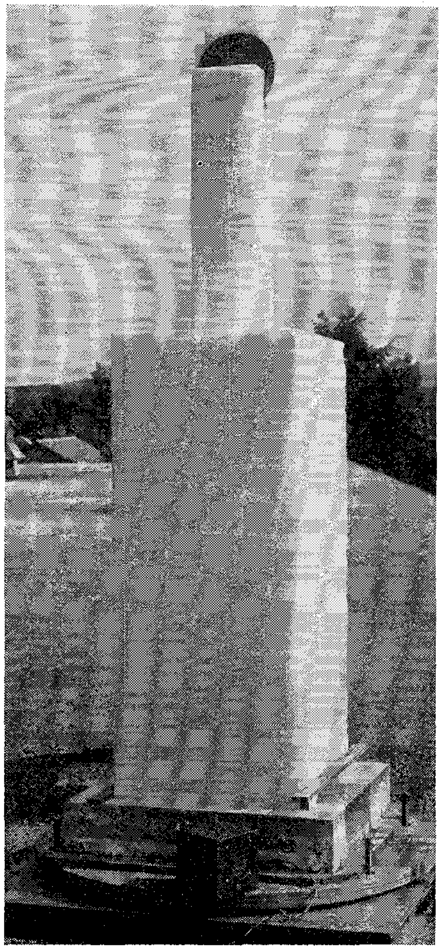

Fig. 2—Polyfoam stand with obstacle on it.

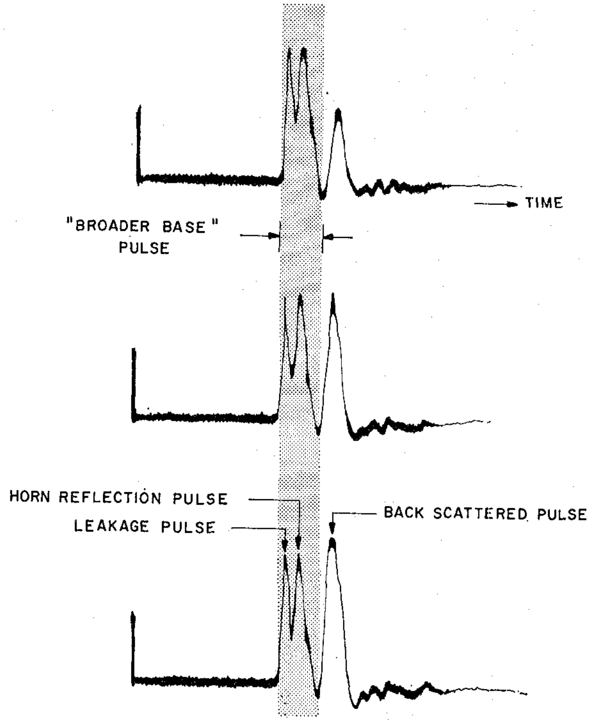

Fig. 3—Oscilloscope traces of broader base pulse and back-scattered pulse (right).

For the present arrangement of $G=350$ and $R=8$ meters, an obstacle with $\sigma=\lambda^2$ will give a back-scattered power of the order of 10^{-9} watt, which is above the noise power level of crystals. However with the limited transmitter power and available detecting system, the minimum detectable signal appears when σ is about $15\ \lambda^2$. The received back-scattered signal is first detected by a video crystal detector and its output is fed into the pulse amplifiers having 60 db of amplification. The output of the amplifier is fed into the 517 Tektronix scope and the height of the pulse on the scope gives a measure of the back-scattering cross section of the obstacle in question. Fig. 3 shows the "broader base" pulse and the back-scattered pulses (right-side ones) for three disks. The broader base pulse is a superposition of two pulses: the leakage pulse due to imperfect decoupling and the reflection pulse due to discontinuities at the ends of the horn. The magnitude of the broader base pulse does not affect that of the back-scattered pulse on the right as long as the former is not large enough to saturate the pulse amplifiers.

During measurements the scattered pulse amplitude for each disk is read from the scope scale which is calibrated against the crystal output in the detecting arm of the hybrid junction T_2. The calibration is made to include both the scope and the pulse amplifiers by using a calibrated pulse attenuator and a millimicrosecond dc pulse generator. The calibrated pulse attenuator has a precision of 5 per cent. Including the errors in scale reading and source amplitude variation, the over-all precision is of the order of 10 per cent. This can be improved by using a precision pulse attenuator, an enlarged scope scale, and a monitored source amplitude. The experimental results are plotted in Fig. 4, normalized to the theoretical value at $ka=10$. These results are in close agreement with the theory. The back-scattering coefficient of the circular disk is closely proportional to the square of its radius at large ka. The results at small ka cannot be obtained with the present available power level. From the data obtained it is seen that the minimum detectable scattering cross section for the present receiving sensitivity and source power level is of about $15\ \lambda^2$. Should a high-power traveling-wave tube amplifier such as the VA-121B (40 watts) be available, scattering cross sections of about λ^2 could be measured by the present system. The smallest back-scattering cross section thus far accurately measured by any method is of the order of $10^{-1}\ \lambda^2$ according to the published data.[3]

The nose-on back-scattering cross section[7] of a perfectly conducting prolate spheroid with a major-axis-to-minor-axis ratio of $a/b=10$ has its first differential minimum at $ka=2.3$. Its back-scattering cross section at this ka is about $10^{-5}\ \lambda^2$. To measure $10^{-5}\ \lambda^2$ cross sec-

[7] K. M. Siegel, F. V. Schultz, B. H. Gere, and F. B. Sleator, "The theoretical and numerical determination of the radar cross section of a prolate spheroid," IRE Trans. on Antennas and Propagation, vol. AP-4, pp. 266–275; July, 1956.

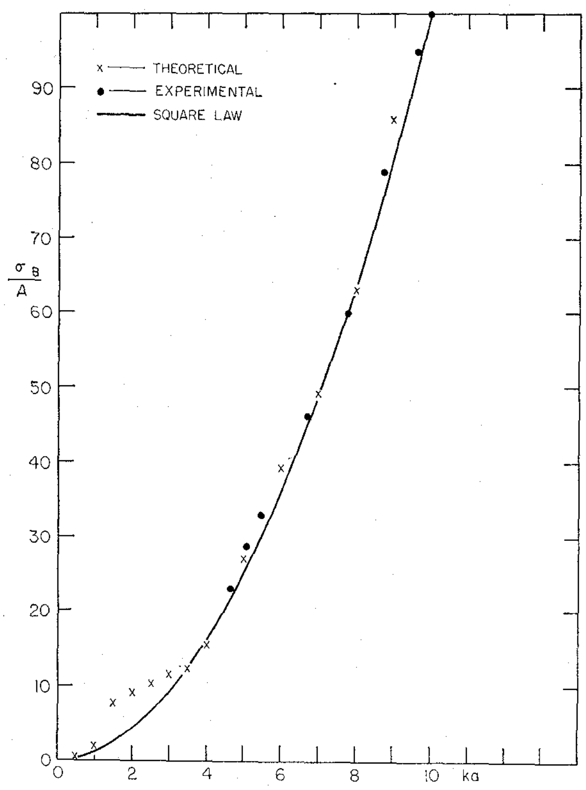

Fig. 4—Back-scattering coefficients of thin circular disks.

tion by the present system of 60 db amplification from the pulse amplifier it is necessary to have an input pulse power of the order of 10 kw. If the receiving pulse amplification can be increased from 60 db to 80 db or 90 db by using a superheterodyne receiving system, the input pulse power level can be reduced to the order of 100 watts or 10 watts. This requires specially designed IF strips with bandwidths at least of the order of 20 mc, since the pulse to be detected has a bandwidth of the order of 50 mc. The use of a specially selected crystal detector and an increase in gain of the horn would also permit an increase in the sensitivity of the system.

If the necessary input power level is so high that the leakage pulse and reflection pulse begin to saturate the amplifiers or burn out the crystal, it is necessary to reduce these pulse levels. The leakage pulse level can only be reduced by increasing the decoupling of the hybrid junction by improving symmetry. To eliminate or reduce the reflections from the discontinuities of the horn, it may be advisable to replace the R_c termination of the hybrid junction T_2 by an identical horn and tuner in order to cancel the reflections by complete symmetry.

In order to reduce possible errors arising from the scale reading of the scope and a nonsquare-law response of the crystal, a precision microwave attenuator can be inserted between the tuner and the matched crystal in the detecting arm of the hybrid junction T_2. In this way all the signals to be detected can be normalized to one convenient scope scale reading by adjusting the precision attenuator, so that the relative attenuator decibel reading is directly proportional to the back-scattered field of the scattering obstacle in question. Th precision of the system as a whole then is limited b that of the precision attenuator.

The theoretical values of the back-scattering coeff cient of thin circular disks are calculated with the hel of the tabulation by Andrejewski[5] and are plotted i Fig. 4. It is seen that the coefficient is closely propo tional to the square of $ka=\gamma$ when γ is larger than 3. I other words, the back-scattering cross section is roughl proportional to the fourth power of the radius of tł disk for γ larger than 3.

The evaluation[5] of the back-scattered field shows th its imaginary part begins to dominate when γ is larg than 3. For an infinitely thin circular disk it is evide that the scattering pattern is symmetrical with respe to the plane of the disk and consequently the scatterir in the forward direction is equal to that in the backwa direction. According to a theorem by Levine ar Schwinger,[8] it is stated that the plane-wave total sca tering cross section of an obstacle is simply proportion to the magnitude of the imaginary part of the forwa scattered field, *i.e.*,

$$\sigma_t = \frac{4\pi}{k^2} \operatorname{Im} F$$

where F is the scattered amplitude in the forward dire tion. From the calculated theoretical values[9] of the tot scattering coefficient (usually called "scattering coe cient") it is found that the total scattering cross secti is closely proportional to the square of the radius of t disk for γ greater than 3. It can be concluded, therefo that for thin circular disks the back-scattering cross se tions are approximately proportional to the square the total scattering cross sections. This apparently pa doxical statement is a consequence of the definition the back-scattering cross section. The back-scatteri cross sections of thin circular disks with γ smaller th 1 are found to be proportional to the sixth power of di radius as shown by various approximations. The tran tional region from a sixth-power law to fourth-pow law lies between $\gamma=1$ and $\gamma=3$. The back-scatteri cross sections of circular disks at small $\gamma=ka$ from (to 3 have been measured by Schmitt[4] using an int ferometer technique. They are in close agreement w the theoretical results.

It is pertinent at this point to make a comparis among the cylindrical obstacles, spherical obstacles, a infinitely thin symmetrical obstacles for both total sc tering cross sections and back-scattering cross sectic in the high-frequency limit. It is to be expected that t total scattering cross sections of cylindrical obstacle

[8] H. Levine and J. Schwinger, "On the theory of diffraction an aperture in an infinite plane screen," pt. 1, *Phys. Rev.*, vol. pp. 958–969; 1948.

[9] C. Huang and R. Kodis, "The Measurement of Aperture Tr mission Coefficients," Cruft Lab. Tech. Rep. 165, Harvard Un 1953.

[10] T. T. Wu. "High frequency scattering," *Phys. Rev.*, vol. 104, 1201–1212; 1956.

spherical obstacles[10] and infinitely thin symmetrical obstacles[9] all approach twice the value obtained by geometrical optics. It is found, however, that the back-scattering cross sections of circular cylinders[3] are $\pi/2$ times the geometrical area, those of spheres[11] unity times the geometrical area, and those of thin circular disks square of the geometrical area.

Conclusion

It is concluded from this preliminary investigation that the time-separation or microwave-pulse method of back-scattering measurements can yield accurate results for three-dimensional obstacles of very small scattering cross section and arbitrary shape provided that a judicial choice and design of each component part of the system is made. Thus it supplements the frequency separation method used by Tang[3] for two-dimensional obstacles.

[11] A. Aden, "Electromagnetic Scattering from Metal and Water Spheres," Ph.D. Dissertation, Harvard Univ.; 1950.

Acknowledgment

The author is indebted to Prof. R. W. P. King for his encouragement during the course of the investigation. He also wishes to express his appreciation to Dr. H. Schmitt for his help on numerous occasions.

On Network Representations of Certain Obstacles in Waveguide Regions*

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Summary—**Network representations for a class of obstacles in waveguide regions when the diffraction problem is of a vector type can be obtained by the use of *E*- and *H*-type modes. The special properties of these modes are discussed and highlighted by an example involving the network representation of a periodic strip grating in free space for oblique incidence. Transformations relating the different networks based on various modal representations in rectangular coordinate systems are also discussed.**

I. Introduction

The problems of the diffraction of electromagnetic waves by obstacles in waveguide or free space are, in general, vector problems. However, in the case of "two-dimensional" obstacles such as the perfectly conducting half plane, infinite periodic gratings, or the infinite circular cylinder in free space, the vector diffraction problem may be decomposed into two independent scalar problems. The same is true in the case of certain structurally similar obstacles in rectangular and parallel plate waveguide. Such decompositions have been employed, for example, by Heins[1] in treating the diffraction of a dipole by a perfectly conducting half plane, and by Levy and Keller[2] in their discussion of diffraction by finitely conducting cylinders at oblique incidence.

In this paper it is shown that modal techniques leading directly to network representations may be employed systematically in the solution of such problems. When the attempt is made to base this approach on the familiar E and H modes propagating perpendicular to the symmetry axis, the desired separation into scalar problems is not possible. On the other hand, the separation into the simpler scalar problems can be effected by appealing to an expansion of the fields in terms of an appropriate alternative set of orthonormal modes. These modes also make it possible to obtain the network representations of problems involving arbitrary angles of incidence directly from the results of the corresponding, strictly two dimensional (incident vector perpendicular to obstacle axis) problems. The matrix relations derived here, which relate the networks based on these modes to networks based on standard E and H modes, further increase the area of applicability of the network solutions.

The modes employed here, which form a complete orthonormal set of vector modes, are designated as the E- and H-type modes. They differ from the familiar H and E modes in that they are characterized by the vanishing of a *transverse*, rather than a longitudinal, field component. To effect the separation into two scalar problems, the modes are chosen such that one sub-set (E-type) has no component of the magnetic field parallel to the axial direction of the "two-dimensional" obstacle, while the second sub-set (H-type) has no com-

* Manuscript received by the PGMTT, July 7, 1958; revised manuscript received, November 4, 1958. The research reported was conducted under Contract AF-19(604)2031, sponsored by the A.F. Cambridge Res. Center, Air Res. and Dev. Command.
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[1] A. E. Heins, "The excitation of a perfectly conducting half-plane by a dipole field," IRE Trans. on Antennas and Propagation, vol. AP-4, pp. 294–296; July, 1956.
[2] B. R. Levy and J. S. Keller, "Diffraction by a Smooth Object," Inst. Math. Sci., New York Univ., N. Y., Res. Rep. EM-109; December, 1957.